# Measurement-Based Replanning of Cell Capacities in GSM Networks

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Due to the scarcity of the spectral resources and mobility of the portables, the call attempts may be blocked during call initiation or terminated during the hand-off process. When the blocking ratio exceeds some grade of service level, the capacity of the congested cell must be replanned using the call attempt data. However, most of the time, the measurements are inflated by the redials and the retrials. During the replanning process, the first step should be to calculate the effective load from the measured cumulative call statistics. In this paper, we provide simple-to-implement analytical models that compute the average number of retrials and redials per original call attempt using only the operations and maintenance center (OMC) measurements. The effective load is then determined through appropriate scaling. That way, unnecessary waste of channels to sustain retrials and redials is avoided during the cellular capacity replanning phase.

**Keywords:** Replanning, retrial, redial, teletraffic

## 1. Introduction

Wireless telephony systems rely on teletraffic or trunking theory to accommodate a large number of users in a limited spectrum while providing a grade of service (GoS). In cellular radio systems, the concept of trunking allows the users to share the relatively small number of channels in a cell by providing on-demand transmission access to each user from a pool of available channels [16]. Trunking exploits the statistical behavior of the users. Due to the scarcity of the spectral resources, there is a trade-off between the number of available channels and the likelihood that a particular caller finds no available channel during the peak calling period. When a particular user requests service and all the radio channels are occupied, access to the system is denied and the user is blocked. To overcome call congestion by increasing the overall channel utilization, Eklundh introduced the directed retrial concept where call set-up attempts search for free radio channels in the neighboring cells upon facing blocking by the associated base station [6]. Deriving from Eklundh's original idea, the Global System for Mobile Communications (GSM) standard presents the cellular operators with the retrial option, where a blocked call may be automatically reattempted for a preset maximum number of times [14]. The maximum retrial count is typically set between three to six by the GSM operators, with no control on the part of the customer. The redials, on the other hand, are call attempts created by the users when they are blocked. Furthermore, a redial may also trigger its own retrials. The initial dimensioning of cellular mobile networks is based on rough estimations of traffic parameters [20], and it may not satisfy the required GoS figure. Consequently, cellular resources must be replanned when congestion is observed. In contrast to the initial dimensioning, this time the network engineers are equipped with the call attempt measurements. A crucial portion of this procedure entails the calculation of the effective load by extracting the excess load created by retrials and redials from the call attempt measurements. In this paper, we present a recipe for the network engineer to calculate the effective load from the call attempt measurements. That way, channel resources are not wasted to sustain retrials and redials during the cellular capacity replanning phase.

In a GSM network, the operations and maintenance center (OMC) measurements contain cumulative call attempt data per cell for e.g. 30 or 60-minute periods, and individual attempt information is not available. Moreover, the OMC data may not include the dropped hand-off attempt counts. The OMC measurements reflect original calls, retrials and redials, all as independent call attempts. Consequently, during the replanning stage, the measured (and inflated) load cannot be directly used. The retrial and redial load must be extracted from the measurements to obtain the effective load which can then be employed in capacity replanning. If the retrials and redials are ignored, they degrade the quality of service since a snowball effect occurs under overload conditions [19]. As an illustration of the direness of the problem, a GSM operator in Turkey recorded a traffic of 207.55 Erlangs in 30 minutes on a certain day in 1999 in Istanbul, ostensibly producing blocking ratio of 86.4%. Applying the Erlang-B model [20,17,1] for instance, the required number of channels for this cell is 222 to carry the offered load with a GoS of 2% blocking. That many channels may not even be available to a single operator. The measured load clearly includes the retrials and redials, and even if 222 channels were assigned to the cell, they would be underutilized because they are not truly needed to sustain the original call traffic. While it is obvious that call statistics must be purged from the retrials and redials, there is no indication in the measurements to identify an individual call as an effective, retrial or redial call. Consequently, the GSM operators need to be equipped with an analytical model which produces the effective load from the measurements. Notice that when the network is designed to support the effective call demands, the number of retrials and redials will diminish, and the system performance will be in the neighborhood of the expected GoS figure. The redial problem in cellular mobile networks is a well-studied subject. In [12], the extended Erlang-B method which assumes that callers redial with a probability and the entire traffic stream is a Poisson process is investigated. Carothers *et al.* propose a model to find the mean number of redials where users are unrealistically patient, redialing infinitely many times [3]. In a similar model, the users redial with a probability after waiting for a period that is exponentially distributed [19]. However, the automatic retrial facility is not addressed in conjunction with redialing in published work [3,13,19,11,7,22,5,6,18].

Furthermore, in previous work, the problem is not formulated in a way that would remove the GSM operators' ennui. In particular, network engineers ask for an easy-to-do procedure that will take the available OMC data as input with no further computational machinery. In this paper, we propose simple, integrated models that provide a solution to the retrial and redial phenomena. The models accept the measured OMC statistics as the input parameters to find the expected number of retrials and redials per effective call attempt. Our straightforward recipe then uses this expectation to scale down the recorded number of attempts to the effective quantity. The latter subsequently gives the effective load together with the mean channel holding time. Finally, the capacity replanning can be carried out according to the effective load. In the next section, we present the analytical models. In Section 3, the analytical models are validated using the simulation results, and finally in Section 4, conclusions are drawn.

## 2. ANALYTICAL RETRIAL AND REDIAL MODELS

Cellular operators rely on the measured call attempt data for the cell capacity replanning process. These measurements are comprised of the following:

(i) total attempt count in 30- or 60-minute periods; (ii) total blocking ratio; (iii) hand-off-to-total attempt ratio; (iv) mean channel holding time; (v) the number of available channels during data collection. The missing pieces of important information are (i) the effective load; (ii) individual hand-off and fresh call blocking ratios (for some OMCs); (iii) time-stamped call arrival information. Time-stamped call statistics can be collected through protocol analyzers connected to the base stations. However, the necessary field operation is difficult and time-consuming considering the hundreds of cells that exist in big cities. In

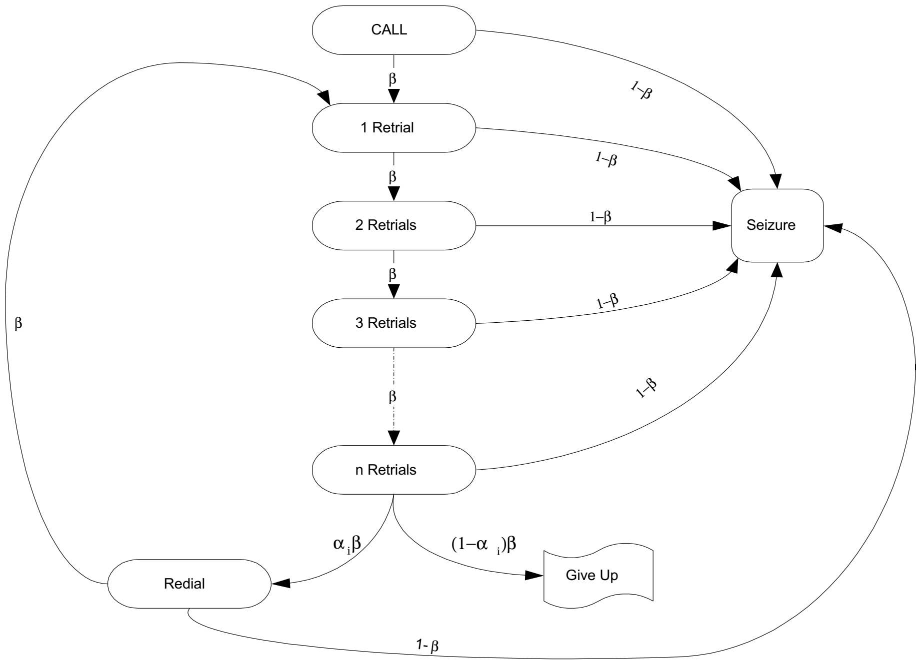



Figure 1. The state transition diagram of an individual call connection request in the Generalized Retrial/Redial Model.

this section, we develop models that remedy the replanning problems by just using the very same inadequate OMC measurements. In particular, we present the analytical models that accommodate the retrials and redials together and produce an estimate of the expected number of retrials and redials per call. This quantity is then used to refine the measured total load in order to obtain the effective traffic load, based on which the capacity replanning can be done more accurately.

The generic call set-up process, the states of the models and the transition probabilities are shown in Figure 1. The process starts at the *Call* state and stops either in the *Seizure* state, or the *Give Up* state. The intermediate states represent the retrials or redials. A stationary blocking ratio $\beta$ of the number of total blocked attempts to total attempts is assumed to act throughout the retrials. This ratio is determined from the adulterated OMC measurements. (The assumption that $\beta$ is approximately invariant to retrials is forced by the unavailability of intermediate OMC data, and its validity is investigated in the sequel.) Each call attempt, as well as the redials, bring along a maximum of $n$ retrials. Upon failure of the $i-1$st call attempt, including all the associated $n$ automatic retrials, the caller either redials with probability $\alpha_i$ (thereby reinitiating the entire call procedure with retrials), or quits with probability $1-\alpha_i$. The model, which we call the generalized retrial/redial model (GRRM) assumes at most $m$ redials per person, and ignores the extremely impatient customers as outliers.

Let $N_R^{\mathrm{GRRM}}$ denote the expected number of retrials and redials per original fresh call. Then, based on the transitions depicted in Figure 1,

$$N_R^{\mathrm{GRRM}} = \sum_{i=0}^{m} \left\{ \left\{ \sum_{j=0}^{n-1} [i(n+1)+j]\beta^{(i+1)n+j}(1-\beta) \right.\right.$$

$$\left.\left. +[(i+1)n+i]\beta^{(i+1)n+i}(1-\alpha_{i+1}\beta) \right\} \xi_i \right\} \quad (1)$$

where

$$\xi_i = \begin{cases} 1 & \text{if } i=0, \\ \prod_{k=1}^{i} \alpha_k & \text{if } i=1,\dots,m. \end{cases} \quad (2)$$

Thus, for every call attempt initiated by a customer, $N_R^{\mathrm{GRRM}}$ additional retrials and redials take place on average. Due to the difficulty in finding a proper redial probability distribution, GRRM can be simplified by assuming a constant redial probability. For a uniform redial probability distribution, where $\alpha_i = \alpha, i = 1,2,\dots,m$, we have $N_R^{\mathrm{GRRM}} = N_R^{\mathrm{URM}}$ where URM stands for uniform redial model. Subsequently, the expression in equation (1) becomes

$$N_R^{\mathrm{URM}} = \sum_{j=0}^{m} \left\{ \left[ \sum_{i=0}^{n-1} (j(n+1)+i)\beta^{(jn+i)}(1-\beta) \right] \right.$$

$$\left. +((j+1)n+j)\beta^{(j+1)n}(1-\alpha\beta) \right\} (\alpha\beta)^j$$

$$= \frac{1 - p_b^{m+1}}{1 - p_b}\left[p_b + \frac{\beta(1-\beta^n)}{1-\beta}\right] - (m+1)(n+1)p_b^{m+1}, \tag{3}$$

where $p_b = \alpha\beta^{n+1}$ is the probability of redialing after the original call and its retrials are all blocked. To simplify URM further, the maximum redial count ($m$) can be omitted by assuming that the users redial till they get a channel or give up redialing. The infinite redial assumption produces optimistic results since the mean number of retrials/redials per effective call increases. However, the network engineer can apply this model, called the infinite redial model (IRM), more easily by getting rid of the maximum redial count. As $m \to \infty$, the expression in equation (3) takes the form in equation (4).

$$N_R^{\mathrm{IRM}} = \lim_{m\to\infty} N_R^{URM} = \frac{\alpha\beta^{n+1}}{1-\alpha\beta^{n+1}} + \frac{\beta(1-\beta^n)}{(1-\beta)(1-\alpha\beta^{n+1})}. \tag{4}$$

Including the original call as well, the following scaling gives the expected effective load ($C_E$):

$$C_E = \frac{C_T}{N_R^{Model} + 1},\ where\ Model = IRM,\ URM\ or\ GRRM,$$

where $C_T$ is the measured total load, which contains retrials and redials. After calculating the effective load, the replanning activity can be performed more accurately. Valuable insight can be attained by evaluating the limiting values of $N_R^{\mathrm{IRM}}$.

$$\lim_{\beta\to 1} N_R^{\mathrm{IRM}} = \frac{n+\alpha}{1-\alpha}. \tag{5}$$

As the blocking ratio $\beta$ approaches unity, the retrials and redials are guaranteed to face blocking. Hence $n$ sure retrials occur per dial attempt. The expression on the right-hand side of equation (5) represents the number of retrials, which is $n$, and redials until the caller gives up dialing and terminates the call set-up process. If, in addition, $\alpha = 0$, then $N_R^{\mathrm{IRM}} = n$, as intuition suggests.

$$\lim_{\alpha\to 0} N_R^{\mathrm{IRM}} = \frac{\beta(1-\beta^n)}{1-\beta}. \tag{6}$$

In the absence of redials, equation (6) is simply the expected number of retrials of the original call.

$$\lim_{\alpha\to 1} N_R^{\mathrm{IRM}} = \lim_{n\to\infty} N_R^{\mathrm{IRM}} = \frac{\beta}{1-\beta}.$$

Since the caller redials with probability one, infinitely many retrials occur, and the first equality holds. The expression on the right-hand side is the ratio of the blocking and success probabilities, giving the average number of blockings (hence retrials) per call set-up. Thus, the asymptotic effective load as $n \to \infty$ is

$$\lim_{n\to\infty} C_E = (1-\beta)C_T.$$

When we omit the retrials, only redials will contribute to ($N_R^{\mathrm{IRM}}$), and equation (4) reduces to

$$\lim_{n\to 0} N_R^{\mathrm{IRM}} = \frac{\alpha\beta}{1-\alpha\beta}, \tag{7}$$

which was first developed in [3].

## 3. Separated Hand-off Model

When a mobile station moves into a different cell during a connection, the call should be handed off to the new cell. A new channel is to be allocated to the connection in the new cell and the previous channel is to be released. During the analysis of the retrial and redial phenomena in GSM networks, the hand-off calls should be modelled separately from the fresh calls, because the hand-offs are not subject to redials. The retrial schemes for both fresh and hand-off calls are the same as explained in the previous section [14]. If a hand-off call is dropped, the user may generate a call to continue with the conversation. The new call is considered as a fresh call in the new cell; but not a redial of the hand-off call. This type of calls are recorded by the fresh call counters. Thus, the hand-off call counters do not register the fresh calls generated after the dropped hand-off calls, which are called as the hand-off feedback calls throughout this paper. The crucial point in this operation is that the user might generate the new fresh call in another cell after the call drops. The reason for this situation might be the variations in power level measurements by the mobile stations that occur due to user mobility. In view of these observations, GRRM is revised to include the hand-offs in the separated hand-off model (SHM). Figure 2 depicts the separation of hand-offs from fresh calls. The connection set-up starts from the *Fresh Call* state and the hand-off starts from the *Hand-off Call* state. The fresh calls are retried and redialed upon being blocked, while the hand-off calls are merely retried up to a specific limit defined by the GSM operator. The retrials scheme is identical for both for fresh and hand-off calls, and it is represented by the maximum retrial count ($n$) in the model. The stationary blocking probability $\beta$ is defined as in GRRM. In Figure 2, the fresh call attempts that are a result of hand-off forced termination are shown by a feedback loop. The hand-off feedback probability $\alpha_H$ is a small value in practice because the users are likely to have crossed over to a neighboring cell by the time they redial. Considering the transitions shown

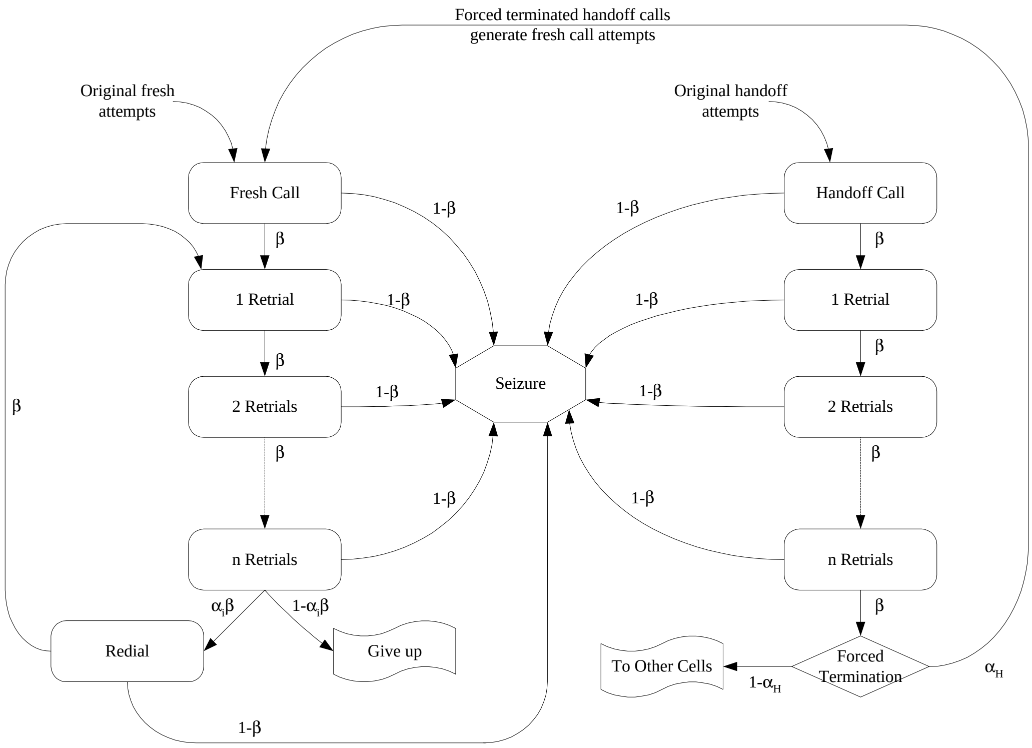



Figure 2. The separated hand-off model.

in Figure 2, the mean number of retrials and redials generated by a fresh call ($N_F^{\text{SHM}}$) is calculated as follows.

$$N_F^{\text{SHM}} = \sum_{i=0}^{m} \Bigg\{ \Bigg\{ \sum_{j=0}^{n-1} [i(n+1)+j]\beta^{(i+1)n+j}(1-\beta)$$

$$+[(i+1)n+i]\beta^{(i+1)n+i}(1-\alpha_{i+1}\beta) \Bigg\} \xi_i \Bigg\} \tag{8}$$

where $\xi_i$ is defined in equation (2). The mean number of retrials by the hand-off calls ($N_H^{\text{SHM}}$) can be calculated as follows:

$$N_H^{\text{SHM}} = \sum_{i=0}^{n-1} i\beta^i(1-\beta) + n\beta^n = \frac{\beta(1-\beta^n)}{1-\beta}$$

Assume that in a given observation interval, the total load is $C_T$, and $f$ per cent of the total load $C_T$ is the hand-off call and retrial load. If we denote the effective fresh call load by $C_F$ ($C_F$ includes the hand-off feedback call load), the effective hand-off call load by $C_H$ and the hand-off feedback load by $C_{HF}$, then

$$C_F = \frac{(1-f)C_T}{N_F^{\text{SHM}}+1},$$

$$C_H = \frac{fC_T}{N_H^{\text{SHM}}+1},$$

$$C_{HF} = \beta^{n+1}\alpha_H C_H.$$

If the cell capacity was planned adequately to handle the attempts for a given GoS, then the hand-off feedback calls and their retrials and redials would not be generated. Thus, the hand-off feedback load should be deducted from the effective load. The effective load $C_E$ can thus be calculated as

$$C_E = C_F - C_{HF} + C_H. \tag{9}$$

The capacity planning of the cell can be done more accurately with this refined quantity.

## 4. Validation of the Models

Three types of simulation models are devised to validate the analytical models. Unlike the analytical models, where no constraints are stipulated on the channel holding times and the interarrival times of the fresh calls, the simulations are carried out according to the Erlang-B assumptions. The Erlang-B formulation is the industry standard for capacity planning among the GSM operators [14,20,17,21]. Furthermore, most researchers use the exponential distribution assumption for the interarrival and channel holding times [3,9,2,4,15,10]. All simulations are performed for 30 minutes following a warm-up period. The time-to-redial is exponentially distributed with a mean of two seconds in simulations. These interretrial delays are not represented in the analytical models. The simulation types are described as follows.

**Type I simulations:** Analytical models are applied to the output of the simulations instead of the OMC measurements. The effective load and the required number of channels produced by the analytical models are compared to the ones obtained in the simulations.

**Type II simulations:** The real OMC measurements obtained from a GSM operator in Turkey (Table 1) are used. However, these measurements lack the effective number of call attempts, and as a result, the analytical models are applied to the real data. The effective number of call attempts in 30 minutes are calculated, and the resulting mean interarrival time of the fresh calls is supplied as an input parameter to the simulator. Afterwards, the real blocking ratio and the total load are compared with those of the simulation.

**Type III simulations:** The analytical models are applied to the real data (see Table 1) to find the effective load and the mean interarrival time of the fresh calls. The Erlang-B model takes the effective load to determine the required number of channels for an industry standard GoS of 2% blocking [14,8]. Afterwards, for a cell which has that many channels, we simulate the retrials and redials with the mean interarrival time of the effective calls. Finally, we compare the resulting blocking ratio with the 2% blocking since the required number of channels were calculated for this GoS figure. For blocking probabilities which are greater than 2%, we run the simulations by incrementing the channel capacity of the cell until we reach a blocking ratio lower than 2%, and then analyze the mismatch.

While they increase our confidence in the analytical models, Type I simulations are not alone sufficient since the blocking probability and the total number of attempts are simulation outputs. Thus, Type II and III models are developed, where each simulation is rerun for 10 times and averaged. The simulations are carried out for 30 minutes. During the experiments, thirteen different real data-sets containing busy hour statistics belonging to cells with different blocking ratio are used. Type II and III simulations are performed with real OMC measurements obtained from a GSM operator in Turkey. The hand-off-to-total load ratio ($f$), the number of channel in the cell ($C$), the mean channel holding time ($\mu$), the blocking ratio ($\beta$) and the total load ($C_T$) are listed in Table 1. The

Table 1
Real traffic measurements over 30 minutes collected by a GSM operator in Turkey.

| Data-set | $f$ | C | $\mu$ (sec) | $\beta$ | $C_T$ (Erl) |
|---|---|---|---|---|---|
| 1 | 0.32 | 28 | 24.90 | 0.12 | 24.24 |
| 2 | 0.36 | 36 | 32.00 | 0.20 | 37.80 |
| 3 | 0.54 | 28 | 27.20 | 0.33 | 37.72 |
| 4 | 0.53 | 28 | 30.30 | 0.38 | 40.69 |
| 5 | 0.55 | 36 | 36.3 | 0.48 | 65.60 |
| 6 | 0.61 | 36 | 38.2 | 0.55 | 77.14 |
| 7 | 0.66 | 28 | 28.40 | 0.57 | 61.38 |
| 8 | 0.71 | 28 | 28.60 | 0.61 | 68.78 |
| 9 | 0.68 | 36 | 41.8 | 0.67 | 101.55 |
| 10 | 0.80 | 28 | 27.90 | 0.76 | 115.26 |
| 11 | 0.83 | 28 | 31.00 | 0.81 | 147.39 |
| 12 | 0.86 | 28 | 30.70 | 0.86 | 196.33 |
| 13 | 0.86 | 28 | 31.80 | 0.86 | 207.55 |

measurement duration is 30 minutes. When the blocking ratio is high, the total load is enormously large. Replanning activity according to this measured total load produces an exorbitant channel requirement. In Type III simulations, this large value of total load is normalized to the effective load using the analytical models, and validated with the simulations.

### *4.1. Sample Redial Distribution*

For the application of GRRM and SHM, one has to identify the exact redial behavior of the customers in the cell of interest. That is, the precise distribution of the redial probability should be used. In a survey conducted among 45 students in the Computer Engineering Department of Boğaziçi University, the redial histogram shown in Figure 3 is obtained. In this histogram, the number of redials turned out to have a mean of 2.47 and a median of 2. Since the probabilities $\{\alpha_i\}_{i=1}^{m}$ are conditioned on the fact that the subscriber has already redialed and failed $i-1$ times, $\{\alpha_i\}_{i=1}^{m}$ forms a decreasing sequence. In particular, the probabilities listed in Table 2 are obtained from Figure 3, and used in the ensuing numerical evaluations. The state transition probabilities shown in Table 2 are the conditional probabilities of redialing.

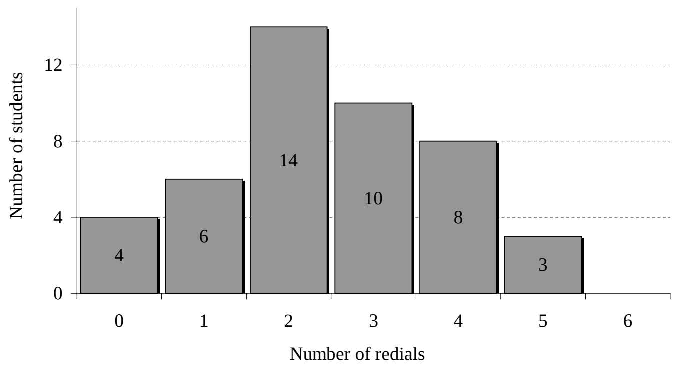



Figure 3. The redial histogram obtained in a survey conducted among 45 students in the Computer Engineering Department of Boğaziçi University.

Table 2
The redial probabilities obtained from the histogram.

| Redial probability | Value |
|---|---|
| $\alpha_0$ | 0.0889 |
| $\alpha_1$ | 0.9111 |
| $\alpha_2$ | 0.8537 |
| $\alpha_3$ | 0.6000 |
| $\alpha_4$ | 0.5238 |
| $\alpha_5$ | 0.2727 |

### *4.2. Type I Simulation Results*

In this section, Type I simulation results are presented. The simulator output is used instead of the measurements when applying the analytical models.

#### *4.2.1. IRM and URM Results*

The simulation duration is 30 minutes. The delay between retrials is 0.96 seconds (typical in GSM networks) and the time-to-redial parameter is chosen to be exponentially distributed with a mean of 2 seconds. The mean interarrival times are adjusted to obtain a total attempt count of 10000 in 30 minutes. The blocking ratio and the total attempt count observed in the simulations and the input parameters to the simulations are used to compute the IRM results. The results are listed in Table 3. When the effective loads produced by IRM are compared with those of the simulations, the results are in close agreement. As the maximum retrial count $n$ increases, the total load carries fewer original calls. The same situation is true as the redial probability $\alpha$ increases, because as $n$ or $\alpha$ increases, the retrials and redials make up a larger portion of the total load. Consequently, the mean number of retrials and redials ($N_R^{\text{IRM}}$) increases, and when the total load is divided by a larger $N_R^{\text{IRM}}$, smaller effective load is produced. However, infinite number of redials results in an overestimated $N_R^{\text{IRM}}$, and the computed effective loads are optimistic.

Table 3
Comparison of the effective loads produced with IRM and the simulations.

| $\alpha$ | $n$ | $C_E$ (IRM) | $C_E$ (Sim.) |
|---|---|---|---|
| 0.3 | 4 | 50.97 | 51.91 |
| 0.3 | 5 | 45.44 | 46.84 |
| 0.3 | 6 | 41.43 | 43.04 |
| 0.6 | 4 | 40.94 | 41.53 |
| 0.6 | 5 | 40.88 | 39.13 |
| 0.6 | 6 | 35.26 | 36.73 |
| 0.9 | 4 | 31.96 | 32.20 |
| 0.9 | 5 | 29.95 | 30.38 |
| 0.9 | 6 | 30.57 | 30.89 |

Table 4
Comparison of effective loads produced with URM and the simulations.

| $\alpha$ | $n$ | $C_E$ (URM) | $C_E$ (Sim.) | $C_E$ (URM) | $C_E$ (Sim.) |
|---|---|---|---|---|---|
| | | $m = 2$ | | $m = 3$ | |
| 0.3 | 4 | 50.76 | 52.29 | 49.99 | 51.42 |
| 0.3 | 5 | 45.04 | 47.34 | 45.56 | 47.23 |
| 0.3 | 6 | 42.77 | 44.49 | 42.21 | 44.46 |
| 0.6 | 4 | 43.57 | 43.96 | 42.54 | 42.78 |
| 0.6 | 5 | 40.55 | 40.00 | 38.87 | 40.02 |
| 0.6 | 6 | 38.32 | 39.12 | 36.80 | 39.12 |
| 0.9 | 4 | 43.57 | 37.61 | 37.40 | 34.78 |
| 0.9 | 5 | 39.58 | 36.70 | 34.92 | 33.33 |
| 0.9 | 6 | 36.19 | 33.33 | 32.63 | 33.03 |

In Table 4, the URM results are in agreement with the simulations. Since the delay between the retrials is not implemented in the analytical model, the original calls in URM have an implicit priority over the successor calls. A call entering URM carries out the retrials as long as it is blocked. Due to the lack of delay, the original call or the repetitions of this call get a channel, if any channel becomes available before the user gives up. However, in real operation, the situation exhibits a random character and there is no implicit or explicit priority of the calls according to their arrival sequence. In practice, the successor calls or their repetitions can access a recently released channel before the repetitions of predecessor calls, because the repetitions occur with a delay as modelled in the simulations. The consequence of this dissimilarity is that when the mean inter-arrival time is less than the time between successive retrials (0.96 seconds), there is a bigger difference between the simulations and URM. Considering that the predecessor calls can get a recently released channel before the successor calls, it is possible to deduce that the blocking probability encountered in each state deviates from the total blocking probability during the simulation period.

The assumption in the analytical models is that the blocking probabilities are the same in all states. However, the delay embedded in the simulations causes the state blocking probabilities to vary. Furthermore, blocking ratios at each state change, when the redial probability is changed. The state blocking ratio in the simulations is the ratio of the blocked attempt count to the total attempt count in that state. The states and the transition probabilities are shown in Figure 1. $R_d$ is the redial count of the call, and $R_t$ is the retrial count of the last call either for the original call or for the redials.

Let the absolute percentage deviation (APD) between the state blocking ratios and the total blocking ratio be defined as below.

$$APD = \frac{1}{(m+1)(n+1)} \sum_{i=0}^{(m+1)(n+1)-1} |\beta_i - \beta| \tag{10}$$

where $\beta_i$ is the blocking probability in state $i$ and $\beta$ is the stationary blocking probability obtained in the simulations.

In Table 5, the state blocking probabilities when $\alpha = 0.6,\ 0.9$, $n = 4$, $m = 2$ are given. The total blocking ratio for $\alpha = 0.6$ is 86.82%, with APD = 2.98%. The corresponding difference in the effective loads is 0.39 Erlangs. When $\alpha = 0.9$, the total blocking ratio is 86.88% with APD = 4.22%, and the effective loads differ by 5.96 Erlangs.

Upon studying Table 5, one can conclude that for the same $n$ and $m$, as the redial probability $\alpha$ increases, the APD increases. The increase in APD is

Table 5
The state blocking ratios in URM simulations for $n = 4$ and $m = 2$.

| Redial count $R_d$ | Retrial count $R_t$ | $\beta_i$ $\alpha = 0.6$ | $\beta_i$ $\alpha = 0.9$ |
|---|---|---|---|
| 0 | 0 | 79.10 | 72.68 |
| 0 | 1 | 87.03 | 88.33 |
| 0 | 2 | 89.00 | 88.11 |
| 0 | 3 | 88.30 | 89.83 |
| 0 | 4 | 86.74 | 89.15 |
| 1 | 0 | 89.43 | 88.79 |
| 1 | 1 | 90.98 | 91.36 |
| 1 | 2 | 90.51 | 90.36 |
| 1 | 3 | 89.51 | 92.35 |
| 1 | 4 | 90.46 | 89.76 |
| 2 | 0 | 89.69 | 90.98 |
| 2 | 1 | 89.08 | 91.89 |
| 2 | 2 | 87.74 | 85.29 |
| 2 | 3 | 94.85 | 93.49 |
| 2 | 4 | 89.15 | 92.62 |

due to the fact that the state blocking probabilities deviate from the total blocking probability more. Moreover, as the variance of state blocking probabilities increases, calculated $N_R^{\text{URM}}$ does not reflect the situation experienced in the simulations. The APD values are around 2-4%. Thus, with such small deviations, it is reasonable to assume a stationary $\beta$ value for all state blocking probabilities.

The effect of redial probability ($\alpha$) on $N_R^{\text{URM}}$ ($n = 5, m = 2$) is depicted in Figure 4. As the redial probability increases the deviation of the calculated $N_R^{\text{URM}}$ from the expected $N_R^{\text{URM}}$ (observed in the simulations) increases. The calculated $N_R^{\text{URM}}$ becomes smaller than the expected one, and the effective load hence exceeds what is experienced in the simulations. For relatively high $n$ or $m$, the impact of the redial probability becomes insignificant, since the retrials influence $N_R^{\text{URM}}$ more than the redials. The calculations in URM produce pessimistic results when $\alpha > 0.7$.

Figure 5 demonstrates that, for $\alpha = 0.5$ and $m = 2$, the calculated $N_R^{\text{URM}}$ does not deviate much from the expectation as the maximum retrial count increases.

For $\alpha = 0.5$ and $n = 5$, the impact of the maximum redial count ($m$) on $N_R^{\text{URM}}$ is shown in Figure 6. The difference between the calculated and expected $N_R^{\text{URM}}$ is preserved as the the maximum redial count is varied.

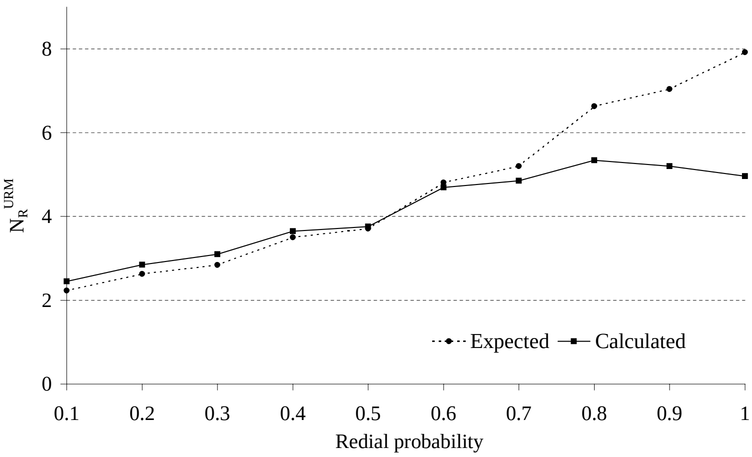



Figure 4. The effect of the redial probability on $N_R^{\mathrm{URM}}$ ($n = 5$, $m = 2$).

In the simulations, the mean interarrival time ($1/\lambda$) is controlled to adjust the effective number of call attempts. As the mean interarrival time increases, the load decreases, because fewer original call attempts are generated. Figure 7 depicts that for $\alpha = 0.6$, $n = 4$ and $m = 2$, the analytical and simulation results converge as $1/\lambda$ grows. The critical value above which we observe agreement turns out to be $1/\lambda = 0.96$ seconds, which is the time between the retrials. The reason for this phenomenon is that when the mean interarrival time is less than the time between the retrials, the successor calls start accessing the recently released channels before the repetitions of the predecessor ones. That is, the successor calls intervene the repetition process of the predecessor calls in the simulations, which cannot happen in the analytical models due to the lack of delay. Consequently, for $1/\lambda < 0.96$ sec, the analytical models produce pessimistic results.

Tables 3 and 4 infer that for low redial probabilities, IRM and URM tend to agree, whereas for high $\alpha$, IRM yields optimistic results. The reason behind this optimism is that when the users are allowed to redial infinitely many times with a large probability, the retrials and redials make up a bigger portion of the total load. Thus, IRM outputs lower effective loads. Since the network planning using pessimistic results is safer, URM might be a better model.

For an accurate application of these two models, the constant redial prob-

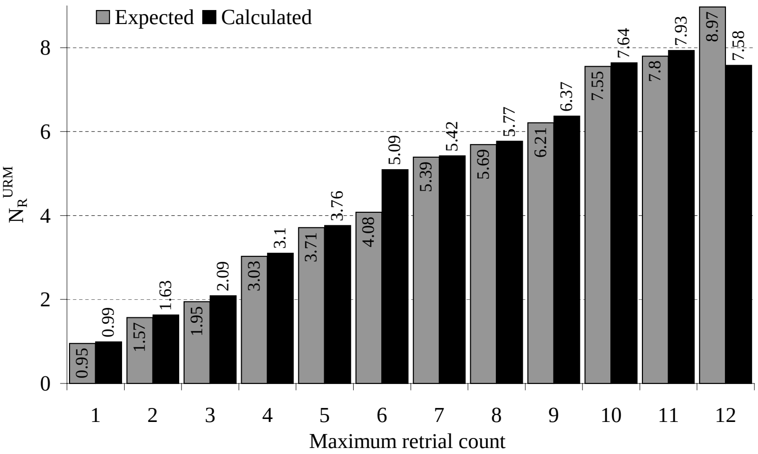



Figure 5. The effect of the maximum retrial count on $N_R^{\mathrm{URM}}$ ($\alpha = 0.5$, $m = 2$).

ability $\alpha$ should be determined carefully. The difference in the effective loads between $\alpha = 0.3$ and $\alpha = 0.9$ can be as much as 20 Erlangs.

*4.2.2. SHM and GRRM Results*

Figure 8 lists the effective load as a function of the mean interarrival time $1/\lambda$, for $n = 4$ and $n = 5$. The mean interarrival time determines the total number of attempts registered in 30 min, and hence the blocking ratio.

The analytical results closely match the simulations. As $1/\lambda$ increases, the effective load decreases, as expected. The choice of $n = 4$ produces fewer retrials/redials on average for low $1/\lambda$, since the network is loaded already and retrials just add to the congestion. In contrast, $n = 5$ produces lower $N_R^{\mathrm{GRRM}}$ when $1/\lambda > 1.5$ sec, implying that retrials tend to be more successful then. The distinction between the two values of $n$ is not only in the corresponding $N_R^{\mathrm{GRRM}}$ values, but also in the total number of call attempts. Thus, the effective load does not necessarily change in the same direction as in $N_R^{\mathrm{GRRM}}$, when going from $n = 4$ to $n = 5$. If one tries replanning disregarding the retrials/redials, the call attempt data used in Figure 8 produce outrageous numbers (as high as 1280 Erlangs!). Both theoretical and experimental GRRM results state that substantially fewer channels are adequate to satisfy the GoS requirement, compared to

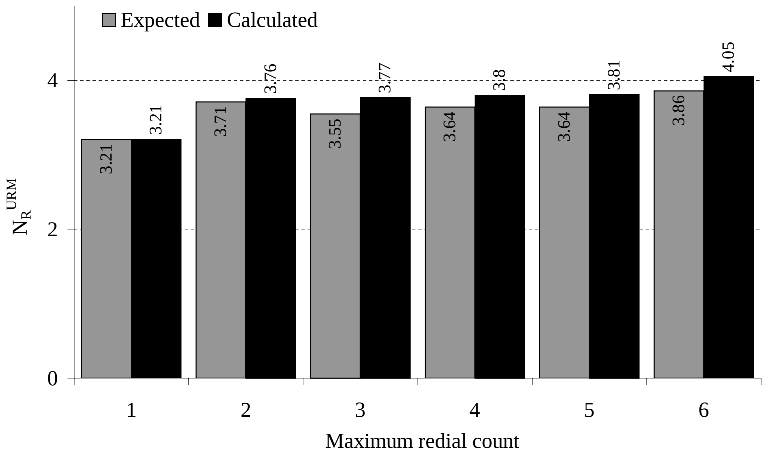



Figure 6. The effect of the maximum redial count on $N_R^{\mathrm{URM}}$ ($\alpha = 0.5$, $n = 5$).

what the direct application of raw numbers suggests.

In Section 4.2.1 we stated that since the delay between the retrials is not implemented in the analytical models, the original calls in URM have an implicit priority over the successor calls. Thus, in Figure 7, when the mean interarrival time is less than the inter-retrial time, there is an disagreement in the results. However, when Figure 8 is analyzed this effect does not seem to exist in GRRM. In the simulations of GRRM, the number of retrials/redials per effective call is larger than that of URM. Thus, the blocking ratio is larger than that of URM, as well. Consequently, the probability of an effective call to find an available channel is very small. Thus, almost all of the effective calls are retried in the simulations of GRRM. As a result, the implicit priority of calls due to arrival sequence that exists in the analytical models takes place in the GRRM simulations naturally. Consequently, the inter-retrial time does not cause a deviation between the analytical and simulation results in GRRM.

Three cases are considered in terms of the mean interarrival times of the effective call attempts:

**Case 1.** The mean interarrival time of the hand-off calls ($\tau_H$) is larger than those of fresh calls ($\tau_F$); that is, the effective fresh calls are more frequent. For this

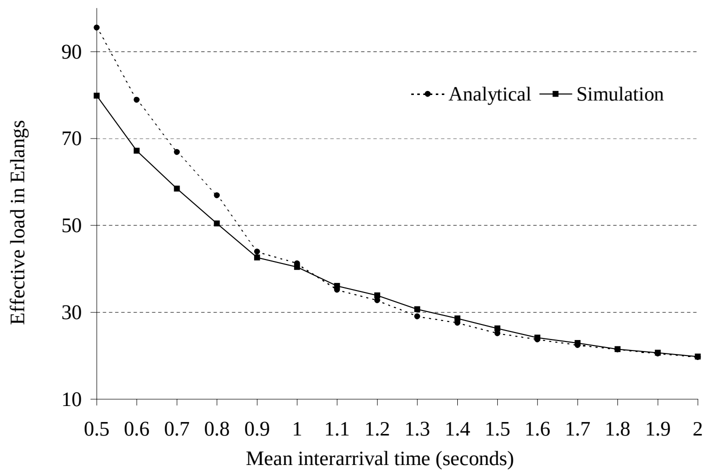



Figure 7. The impact of $1/\lambda$ on the effective load and the required number of channels in URM and the simulations.

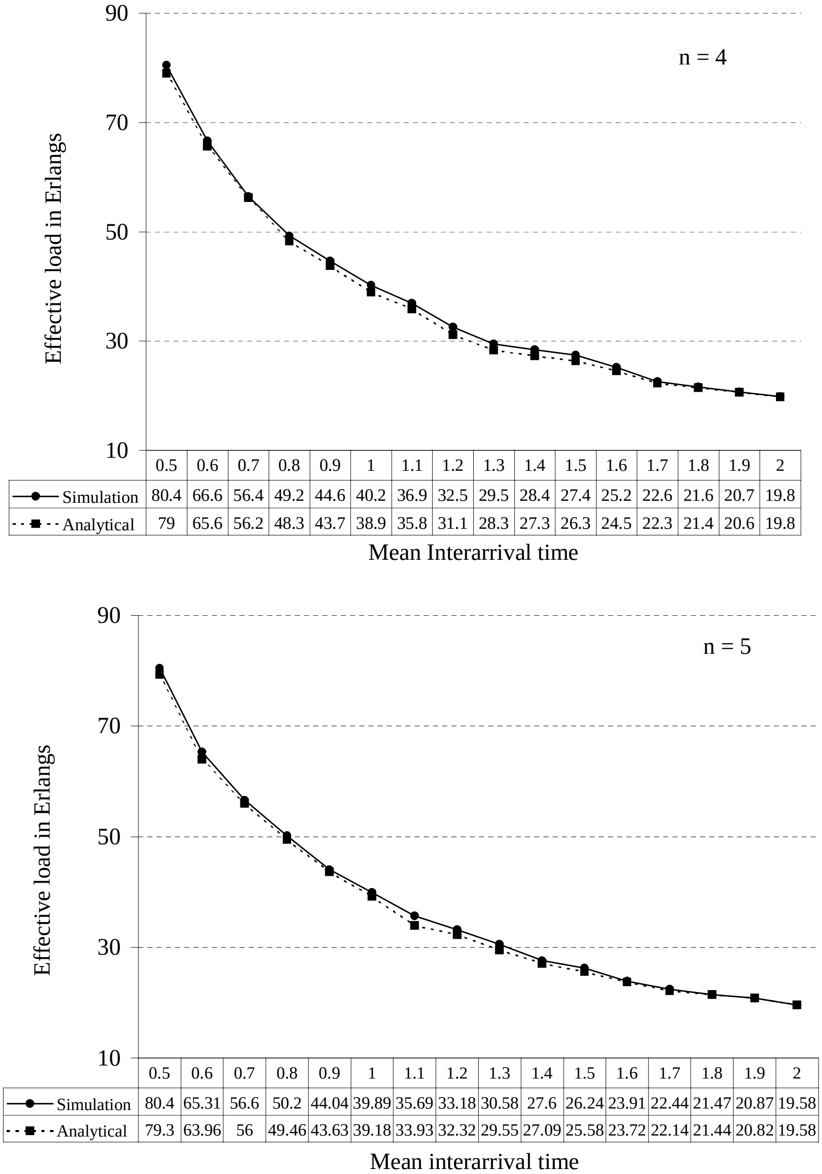
case, $\tau_F = 1$ seconds, $\tau_H = 2$ seconds, $n = 5$.

**Case 2.** The mean interarrival time of the fresh calls is larger than those of the hand-off calls; that is, the effective hand-off calls are more frequent. For this case, $\tau_F = 2$ seconds, $\tau_H = 1$ seconds, $n = 5$.

**Case 3.** The mean interarrival times are equal. For this case, $\tau_F = \tau_H = 1.5$ seconds, $n = 5$.

The total load ($C_T$), the effective load ($C_E$), the handoff-to-total load ratio ($f$) and the blocking ratio ($\beta$) are obtained. Onto these results, the analytical models are applied. The effective loads are tabulated in Figures 9 and 10. When mean the interarrival times decrease, the total attempt count and $\beta$ increase. As $\beta$ becomes larger, the mean number of retrial/redials per effective call grows. A similar set of implications take place when $\alpha_H$ increases.

In Figure 9, the resulting effective loads of SHM are in close agreement with those of the simulation model where the fresh calls are more influential on the resulting total attempt count than the hand-off calls. For example, in Figure 9, for $\alpha_H = 0.5$, the total load including the retrials and redials is 756.38 Erlangs. SHM produces an effective load of 58.94 Erlangs, which is in agreement with




| | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Simulation | 80.4 | 66.6 | 56.4 | 49.2 | 44.6 | 40.2 | 36.9 | 32.5 | 29.5 | 28.4 | 27.4 | 25.2 | 22.6 | 21.6 | 20.7 | 19.8 |
| Analytical | 79 | 65.6 | 56.2 | 48.3 | 43.7 | 38.9 | 35.8 | 31.1 | 28.3 | 27.3 | 26.3 | 24.5 | 22.3 | 21.4 | 20.6 | 19.8 |

| | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Simulation | 80.4 | 65.31 | 56.6 | 50.2 | 44.04 | 39.89 | 35.69 | 33.18 | 30.58 | 27.6 | 26.24 | 23.91 | 22.44 | 21.47 | 20.87 | 19.58 |
| Analytical | 79.3 | 63.96 | 56 | 49.46 | 43.63 | 39.18 | 33.93 | 32.32 | 29.55 | 27.09 | 25.58 | 23.72 | 22.14 | 21.44 | 20.82 | 19.58 |

Figure 8. Analytical and simulation results for $N_R^{\mathrm{GRRM}}$ and the resulting effective load for $n = 4, 5$.

simulation result of 60.27 Erlangs. The huge difference between the total load and the effective loads emphasizes the importance of weeding out the retrial/redials in cellular networks.

Furthermore, when the hand-off feedback probability ($\alpha_H$) is altered, the effective load calculated with SHM does not change much. Figure 9 indicates that if $\alpha_H < 0.3$, the hand-off feedback load can be omitted. Since the mobile station measures the power level gained from several different cells, it is highly probable for a mobile station to camp on a cell other than the one to which the hand-off is attempted after the call drops. Moreover, the dropped attempt can result in a fresh call attempt depending upon the user. Consequently, it is intuitively plausible to conclude that the hand-off feedback probability is not large in practice.

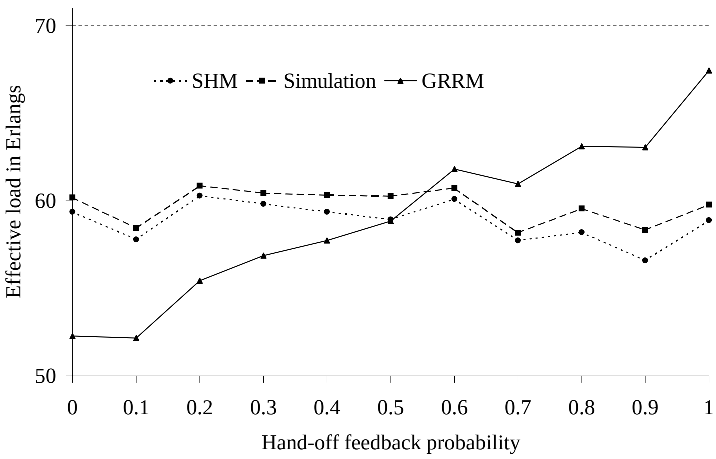



Figure 9. The effective load comparison of SHM with simulations and GRRM for Case 1 ($\tau_H > \tau_F$). (Effective loads are in Erlangs.)

As $\alpha_H$ increases, the total number of attempts, the blocking probability, the mean number of retrials and redials for the fresh calls ($N_F$), and the mean number of retrials for the hand-off calls ($N_H$) all increase. The effect of $\alpha_H$ is more on the total attempt count than its effect on $N_H$ or $N_F$. Consequently, the effective fresh and hand-off call counts both grow. The hand-off feedback count increases with the hand-off count and the blocking probability. Thus the difference of the effective fresh call count and the hand-off feedback call count remains almost constant.

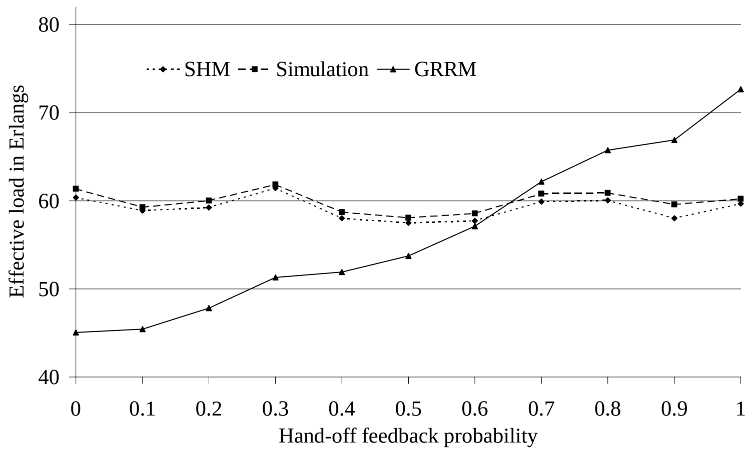



Figure 10. The effective load comparison of SHM with simulations and GRRM for Case 2 ($\tau_H < \tau_F$). (Effective loads are in Erlangs.)

In Figure 9, SHM is compared to GRRM and the simulations. When the fresh calls are more dominant in the total attempt count, SHM acts like GRRM, and thus the difference in the effective loads are not large. However, when the hand-off calls make up a large portion of the total attempt count, the difference becomes higher. Because, in GRRM the hand-off calls are considered like fresh calls, and GRRM explicitly imposes the error of considering the redials of the hand-off calls. In fact, the hand-off calls are not subject to redials. Consequently, for small $\alpha_H$, GRRM produces optimistic results, but as $\alpha_H$ gets larger, GRRM approaches to SHM (see Figures 9,10). For large $\alpha_H$, GRRM starts to produce pessimistic results. Because, when $\alpha_h$ gets larger, the total attempt count increases, consequently the blocking ratio ($\beta$) increases. When the blocking ratio increases, $N_R$ increases. Since the increase in $N_R$ is not as much as the increase in total load ($C_T$), $C_E$ in GRRM becomes larger for large $\alpha_H$. Thus the effective load increases and the results become pessimistic. However, for all cases, when $\alpha_H$ is around 0.6, SHM and GRRM results are in close agreement with those of the simulations. When SHM is compared with the simulations, the same conclusions can be drawn for cases 2 and 3. On the other hand, as the arrival rate of the hand-off calls exceeds the rate of the fresh calls, the hand-off calls are more dominant on the total attempt count, and the difference between SHM and GRRM becomes more pronounced. Thus, the engineer should resort to GRRM as an overcautious solution, if the reliable hand-off statistics are not available.

### *4.3. Type II Simulation Results*

Type II simulations use measurements obtained from a GSM operator in Turkey, and they check how well the simulations model the real operations in a GSM cell. In Table 6, the measured blocking ratios are compared to those generated by Type II simulations. The simulations approximate the real operation good enough when the actual blocking ratio is large, and the analytical models can be applied to the cells where high blocking is observed. In Table 7, the real

Table 6
Blocking ratio comparison between Type II simulations and the analytical models for $n = 4, m = 3, \alpha = 0.6, \alpha_H = 0$.

| Data-set | Actual blocking ratio | IRM | URM | GRRM | SHM |
|---|---|---|---|---|---|
| 1 | 0.12 | 0.16 | 0.20 | 0.18 | 0.18 |
| 2 | 0.20 | 0.31 | 0.28 | 0.31 | 0.30 |
| 3 | 0.33 | 0.49 | 0.52 | 0.61 | 0.46 |
| 4 | 0.38 | 0.51 | 0.49 | 0.60 | 0.46 |
| 5 | 0.48 | 0.61 | 0.57 | 0.70 | 0.52 |
| 6 | 0.55 | 0.61 | 0.61 | 0.72 | 0.56 |
| 7 | 0.57 | 0.60 | 0.57 | 0.67 | 0.55 |
| 8 | 0.61 | 0.63 | 0.62 | 0.71 | 0.61 |
| 9 | 0.67 | 0.65 | 0.64 | 0.66 | 0.61 |
| 10 | 0.76 | 0.76 | 0.75 | 0.79 | 0.74 |
| 11 | 0.81 | 0.81 | 0.82 | 0.81 | 0.81 |
| 12 | 0.86 | 0.85 | 0.86 | 0.85 | 0.85 |
| 13 | 0.86 | 0.87 | 0.87 | 0.86 | 0.87 |

total load is compared with those that result in the simulations. The measured and simulation total loads are in agreement for those data-sets where the blocking ratio is greater than 50%. Hence, the analytical models can be applied to highly congested cells which hold the bulk of the demand. In those data-sets that reflect high congestion, not only the blocking ratio but also the hand-off-to-total attempt ratio increases. Since GRRM does not model the hand-offs separately, it includes the redials of the hand-off attempts in the calculations, whereas in

Table 7
Total load comparison between type II simulation and the analytical models for $n = 4, m = 3, \alpha = 0.6, \alpha_H = 0$.

| Data-set | Actual total load | IRM | URM | GRRM | SHM |
|---|---|---|---|---|---|
| 1 | 24.24 | 25.05 | 26.01 | 25.83 | 25.45 |
| 2 | 37.80 | 42.83 | 40.92 | 43.02 | 42.10 |
| 3 | 37.72 | 47.15 | 50.79 | 62.17 | 45.15 |
| 4 | 40.69 | 49.69 | 47.34 | 60.95 | 45.48 |
| 5 | 65.60 | 85.37 | 76.03 | 109.44 | 67.01 |
| 6 | 77.14 | 84.17 | 84.57 | 119.99 | 74.41 |
| 7 | 61.38 | 62.46 | 58.52 | 77.64 | 55.19 |
| 8 | 68.78 | 68.77 | 67.67 | 88.82 | 66.19 |
| 9 | 101.55 | 95.58 | 91.10 | 97.16 | 86.48 |
| 10 | 115.26 | 112.75 | 107.24 | 128.50 | 105.63 |
| 11 | 147.39 | 144.24 | 152.55 | 145.55 | 142.08 |
| 12 | 196.33 | 189.13 | 197.32 | 177.77 | 189.11 |
| 13 | 207.55 | 211.08 | 216.28 | 203.77 | 206.22 |

reality the hand-off attempts are not subject to redials. Consequently, the blocking ratios obtained in Type II GRRM simulations are larger than those in SHM simulations. This situation has a contrary effect in the analytical model. Since the redials of hand-offs are considered in GRRM, the mean number of retrials and redials ($N_R$) increase and GRRM produces optimistic results. The same results can be drawn by observing the blocking ratios from Table 6.

### *4.3.1. Adjustment of Maximum Retrial Count*

Since the maximum retrial number $n$ is a network parameter, the network operators need a means to adjust it. The analytical models that we propose in this paper are not developed to provide a recipe to adjust $n$ but a recipe to compute the mean number of retrials and redials which should be used to compute the effective load during the replanning process according to the OMC measurements. However, the simulation models can be used to give an insight to the network engineer about what kind of a strategy to be used during the adjustment of the parameter $n$.

In Table 8, the effect of maximum retrial count on the total load $C_T$ and the blocking ratio $\beta$ in URM is shown. In order to find the effective load, URM ($n = 4, m = 3, \alpha = 0.6$) is applied on the real data listed in Table 1. With the obtained effective loads for each data-set, the mean interarrival times are calculated and

Table 8
The effect of the maximum retrial count on the total load and blocking ratio in URM where $m = 3, \alpha = 0.6$.

| | $n=2$ | | $n=3$ | | $n=4$ | | $n=5$ | | $n=6$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| Data-set | $C_T$ | $\beta$ | $C_T$ | $\beta$ | $C_T$ | $\beta$ | $C_T$ | $\beta$ | $C_T$ | $\beta$ |
| 1 | 24.46 | 0.14 | 25.10 | 0.16 | 26.48 | 0.20 | 27.90 | 0.24 | 27.21 | 0.23 |
| 2 | 37.81 | 0.22 | 39.91 | 0.26 | 43.53 | 0.31 | 46.69 | 0.36 | 45.17 | 0.34 |
| 3 | 36.82 | 0.36 | 42.84 | 0.44 | 48.11 | 0.49 | 49.30 | 0.51 | 56.14 | 0.57 |
| 4 | 39.34 | 0.40 | 45.52 | 0.47 | 46.45 | 0.48 | 52.15 | 0.54 | 62.33 | 0.61 |
| 5 | 57.62 | 0.44 | 65.81 | 0.51 | 78.17 | 0.58 | 81.84 | 0.60 | 84.36 | 0.61 |
| 6 | 61.22 | 0.47 | 74.92 | 0.56 | 82.91 | 0.60 | 93.71 | 0.65 | 103.47 | 0.68 |
| 7 | 45.34 | 0.46 | 53.23 | 0.54 | 59.51 | 0.57 | 65.31 | 0.61 | 71.48 | 0.65 |
| 8 | 51.92 | 0.51 | 65.15 | 0.61 | 66.57 | 0.61 | 79.21 | 0.68 | 84.83 | 0.69 |
| 9 | 63.23 | 0.48 | 78.03 | 0.58 | 91.27 | 0.64 | 100.62 | 0.67 | 103.07 | 0.67 |
| 10 | 72.99 | 0.64 | 87.35 | 0.69 | 105.20 | 0.75 | 135.13 | 0.80 | 144.32 | 0.81 |
| 11 | 95.66 | 0.72 | 120.63 | 0.78 | 145.03 | 0.81 | 177.11 | 0.85 | 210.63 | 0.87 |
| 12 | 128.89 | 0.79 | 168.37 | 0.83 | 205.70 | 0.87 | 247.58 | 0.89 | 276.45 | 0.90 |
| 13 | 138.95 | 0.80 | 177.85 | 0.84 | 217.36 | 0.87 | 258.53 | 0.89 | 304.13 | 0.91 |

the URM simulations are carried out with different values of maximum retrial count to obtain the total load and the blocking ratio. As depicted in Table 8, the maximum retrial count has a significant effect on the total load and blocking ratio especially for highly congested cell. However, for cells which are slightly congested, as the maximum retrial count increases, the increase in blocking ratio is more than the increase in the total load.

On analyzing Table 8, it can be concluded that the network will perform better when the maximum retrial count is adjusted dynamically. That is, when the cell is highly congested $n$ should be decreased to provide a better overall GoS and when the congestion level decreases $n$ can be increased to provide a better GoS for the individuals although the overall GoS increases slightly.

### *4.4. Type III Simulation Results*

In Type III simulations, the analytical models are applied to the data-sets listed in Table 1 to find the effective load and the mean interarrival time of the original calls. Erlang-B model is applied to the effective load to determine the required number of channels for a GoS of 2% blocking. Afterwards, for a cell which has that many channels, we simulate the system with the mean interarrival time of the original calls. For blocking ratios that are greater than 2%, the simulations are rerun by incrementing the radio channel capacity of the cell until a blocking ratio lower than 2% is reached, and the mismatch is analyzed. In Tables 9 and 10, the effective required number of channels are tabulated for the analytical models and simulations to provide a GoS less than or equal to 2% blocking. Table 9 verifies that the retrials are predominant on

Table 9
Type III simulation results for IRM and URM.

| Data-set | Effective load (GRRM) | Required channels (GoS=2%) | IRM $n=4, \alpha=0.6$ | | URM $n=4, m=3, \alpha=0.6$ | |
|---|---|---|---|---|---|---|
| | | | Analytical | Simulation | Analytical | Simulation |
| 1 | 24.24 | 33 | 30 | 34 | 30 | 33 |
| 2 | 37.80 | 48 | 40 | 45 | 40 | 45 |
| 3 | 37.72 | 48 | 34 | 37 | 34 | 38 |
| 4 | 40.69 | 51 | 34 | 39 | 34 | 39 |
| 5 | 65.60 | 77 | 44 | 49 | 44 | 49 |
| 6 | 77.14 | 89 | 45 | 51 | 45 | 53 |
| 7 | 61.38 | 73 | 36 | 41 | 36 | 41 |
| 8 | 68.78 | 81 | 37 | 41 | 37 | 41 |
| 9 | 101.55 | 114 | 46 | 51 | 46 | 50 |
| 10 | 115.26 | 129 | 41 | 45 | 41 | 47 |
| 11 | 147.39 | 161 | 44 | 49 | 44 | 49 |
| 12 | 196.33 | 211 | 48 | 52 | 49 | 54 |
| 13 | 207.55 | 222 | 49 | 55 | 50 | 56 |

the total load when the redial probability is not large, since IRM and URM produce similar required number of channels. For the data-sets with a high blocking ratio, the discrepancy between the analytical and simulation results is acceptable, considering the required number of channels for the total load.

Table 10 lists Type III simulation results for SHM, which are pessimistic in comparison to GRRM. Again, GRRM produces optimistic results because it considers the redials of the blocked hand-off calls. The same conclusions are drawn from Type II simulations. The SHM and GRRM results are in agreement when the hand-off-to-total attempt ratio is small. However, when the hand-off attempts become more dominating in the total attempt count, GRRM starts undershooting.

As can be seen in Tables 9 and 10, the mismatch between the required number of channels obtained from the analytical model and the required number of channels to make the blocking ratio smaller than 2% is around 4 to 6. As an example, for the 13th data-set, if the required number of channels is calculated

Table 10
Type III Simulation Results for GRRM and SHM.

| Data-set | Effective load (GRRM) | Required channels (GoS=2%) | GRRM $n = 4$ | | SHM $n = 4, \alpha_H = 0$ | |
|---|---|---|---|---|---|---|
| | | | Analytical | Simulation | Analytical | Simulation |
| 1 | 24.24 | 33 | 30 | 34 | 30 | 33 |
| 2 | 37.80 | 48 | 40 | 44 | 40 | 44 |
| 3 | 37.72 | 48 | 34 | 38 | 34 | 37 |
| 4 | 40.69 | 51 | 34 | 38 | 34 | 38 |
| 5 | 65.60 | 77 | 44 | 50 | 45 | 50 |
| 6 | 77.14 | 89 | 45 | 50 | 46 | 52 |
| 7 | 61.38 | 73 | 35 | 39 | 37 | 41 |
| 8 | 68.78 | 81 | 36 | 42 | 38 | 42 |
| 9 | 101.55 | 114 | 44 | 49 | 48 | 52 |
| 10 | 115.26 | 129 | 38 | 44 | 45 | 50 |
| 11 | 147.39 | 161 | 39 | 44 | 51 | 55 |
| 12 | 196.33 | 211 | 41 | 47 | 60 | 66 |
| 13 | 207.55 | 222 | 42 | 47 | 62 | 69 |

for a GoS of 2% blocking using the total load (207.55 Erlangs), 222 channels should be assigned to the cell. With GRRM, the required number of channels is reduced to 42 from 222, while simulations give 47.

## 5. Conclusions

Since the cellular network operators charge users when the call set-up is successful, the blocking events affect the revenues. The radio channel capacity assigned to each cell should be continuously replanned to ensure a predetermined GoS goal. To that end, the measurements should be refined with the proposed analytical models to obtain the effective load by extracting the retrials and redials which occur due to high blocking ratios.

The significance of these models are in their simplicity and easy-to-use properties. In the literature, there are models proposed to solve the redial problems, but they are very difficult to be used by a network engineer. In this paper, we address not only the redial problem but also the retrial issue, which is defined by the network parameters. Due to the difference in the redial behaviors of the fresh calls and hand-offs, we introduce the hand-off feedback concept. The analytical models in this paper do not assume any distribution about the interarrival and channel holding times. However, we invoked the Erlang-B assumptions in the simulations to validate the analytical models but other distributions could be used for validation. Furthermore, in the analytical models, the delay between the retrials or the redials are not considered. This gives a priority to the calls on the channel utilization according to the arrival sequence; however, in practice, the predecessor call or their retrials do not have any priority over the successor calls. This phenomenon in the analytical models causes the blocking ratio to fluctuate. However, in Type I simulations, we investigated that the stationary blocking ratio is an acceptable assumption since the state blocking ratio deviates from the stationary blocking ratio only around 2-4%. Type II simulations showed that the simulations model the network closely. In Type III simulations, we observed that the analytical models perform well in highly congested cells.

While IRM and URM are tailored to uniform distributed redials, GRRM and SHM are designed to work with any redial distribution. In any case, for the appropriate application of these models, the redial behavior of the subscriber should be determined. However, if this is not possible, IRM or URM can be applied. Furthermore, if the redial count of the users cannot be determined IRM will provide the necessary recipe to calculate the effective load. Finally, if the retrial facility is not implemented in the network, the limit value of IRM can be used.

## Appendix

List of Symbols

| | |
|---|---|
| APD | Average percentage deviation |
| $C$ | Number of channels |
| $C_E$ | Number of effective call attempts |
| $C_F$ | Number of effective fresh call attempts |
| $C_H$ | Number of effective hand-off call attempts |
| $C_{HF}$ | Number of effective hand-off feedback call attempts |
| $C_T$ | Total number of call attempts |
| $f$ | Hand-off to total attempt count ratio |
| $m$ | Maximum redial count |
| $n$ | Maximum retrial count |
| $N_F^{\text{SHM}}$ | Mean number of retrials/redials per effective fresh calls in SHM |
| $N_H^{\text{SHM}}$ | Mean number of retrials/redials per effective hand-off calls in SHM |

| | |
|---|---|
| $N_R^{\mathrm{IRM}}$ | Mean number of retrials/redials per effective call in IRM |
| $N_R^{\mathrm{URM}}$ | Mean number of retrials/redials per effective call in URM |
| $N_R^{\mathrm{GRRM}}$ | Mean number of retrials/redials per effective call in GRRM |
| $R_d$ | Redial count of a user |
| $R_t$ | Retrial count of a user |
| $\alpha$ | Redial probability |
| $\lambda$ | Mean arrival rate of call requests |
| $\beta$ | Blocking probability |
| $\beta_i$ | Blocking probability in $i^{th}$ state |
| $\mu$ | Service rate |
| $\alpha_i$ | $i^{\mathrm{th}}$ redial probability |
| $\alpha_H$ | Hand-off feedback probability |
| $\xi_i$ | $i^{\mathrm{th}}$ redial probability multiplication factor |
| $\lambda_H$ | Hand-off call arrival rate |
| $\lambda_F$ | Fresh call arrival rate |

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